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A B S T R A C T

The Diamondback Moth (*Plutella xylostella*) is a notorious agricultural pest that poses significant challenges to cabbage production. In this study, we formulated and analyzed the deterministic differential equations to capture the infestations dynamics of diamondback moth in a cabbages biomass, taking into account the use of environmentally friendly pesticides. To study its dynamics we computed the threshold number, $\mathscr{R}_*$, based on the pest-free equilibrium point. The results indicate that when $\mathscr{R}_* \leq 1$, the equilibrium point ξ_1 is both locally and globally stable. Conversely, when $\mathscr{R}_* > 1$, the coexistence point becomes globally asymptotically stable. The stability of the equilibrium points were both Locally and globally assessed using Ruth Hurwitz's criteria for local stability and Lyapunov functions for global analysis. A comprehensive numerical analysis was conducted, confirming the substantial support for the analytical findings. Finally, this research suggests that in order to reduce the impact of the diamondback moth, it is necessary to decrease the threshold value smaller than a unity through the adoption of effective inter-cropping techniques and the use of environmentally friendly pesticides.

1. Introduction

Cabbage is among the vegetables with nutritional values such as vitamins, minerals and ascorbic acid to the human body [1]. Cabbages lowers the incidences of getting chronic diseases such as cancer and heart diseases [2]. As agricultural produce, it provides food and income to farmers and foreign currency to the nation. Cabbage grows well in areas with a rainfall of 300–500 mm; a temperature between 16–20 °C; and a soil pH range of 6.0–6.5 [3,4].

In 2020, the global cabbage production reached approximately 71 million metric tons where China accounted for 48% of the total production, while East Africa contributed around 21% [5]. Unfortunately, cabbage crops are susceptible to various pest infestations, including cutworms, cabbage moths, diamondback moths, and plant diseases, which result in significant yield losses.

The Diamondback moth (DBM) scientifically called *Plutella xylostela*, is a highly destructive pest that infests cabbages and other cruciferous vegetables, destroying the folliar tissues and heads of the cabbages [6]. It originated in Europe but, due to its migratory behavior, has spread worldwide, especially in the regions where cruciferous vegetables are grown [6]. DBM is prevalent in various African countries, including Kenya, Egypt, Nigeria, Niger, Tanzania, and Uganda [6–9].

The DBM thrives in areas with temperature ranges between 6 °C and 30 °C [6]. The presence of pests, such as the diamondback moth (DBM), in cabbage production leads to substantial economic losses for farmers. Globally, the management of DBM incurs substantial costs, estimated to be around 4 to 5 billion USD [8,10]. For instance, China incurs an annual cost of USD 0.77 billion to manage DBM [11]; in India, DBM causes an economic loss of 50% of 168 million USD per year [12,13]; and Africa the weekly cost of approximately USD 46,097,772 goes to DBM management [10]. To alleviate the burden of DBM and enhance cabbage production, it is crucial to implement integrated pest control approaches. Integrated pest management strategies effectively suppress pests while minimizing environmental impact. By adopting such approaches, farmers can mitigate the negative effects of DBM infestations and improve cabbage production.

Integrated pest management (IPM) is a strategy used by farmers to control the infestation of pest in the farm by employing multiple pest control strategies [14–20]. These strategies encompass a range of approaches such as Biological control: Farmers utilize biological agents such as natural enemies (predators, parasitoids) to control pest populations [21–23]; Habitat management and cropping practices: Implementing practices like crop rotation, inter-cropping, and maintaining

suitable habitats for natural enemies can help manage pests effectively [24–27]; Synthetic pesticides and chemicals: Carefully selected synthetic pesticides may be used as a part of IPM strategies to control pests when necessary [28–30].

Furthermore, the application of ecological modeling concepts can be valuable in capturing and describing the complex processes within agricultural ecosystems. The models helps to simulate and give the predictions of pest's dynamics, to assess effectiveness of different control measures, and optimize resource allocation for pest management. By integrating ecological knowledge and mathematical modeling, farmers can develop more holistic and effective IPM strategies that are tailored to their specific agricultural ecosystems. This approach can contribute to sustainable pest management practices and promote the overall health and productivity of agricultural systems [31].

Different scholars employed statistical approaches to study the infestations dynamics of the DBM [32,33]. In a similar study, Dymex modeling approach was applied to capture the infestation dynamics and control of the pest by involving the climate, cropping patterns, and natural enemies [26]. The study reveals that the use of biological control techniques is significant for managing the diamondback moth. The review that focused on application of bio-control agents for DBM management.suggests that the pest is controlled through the deployment of beneficial natural enemies [34]. On the other hand, another study examines the phenology of the Diamondback moth and the monitoring methods. This study reveals that the Diamondback moth is managed through the application of pheromone traps [35]. Previous studies have made valuable contributions to understanding the management of diamondback moth infestations in cabbage biomass through modeling and statistical approaches. However, these studies have not fully incorporated the application of selected pesticides with minimal impacts to beneficial insects and the environment.

In this study, we address such a limitation by developing a comprehensive model system, allowing for a more accurate representation of the infestation dynamics on cabbages. Our model system integrates the use of selected pesticides that have minimal impacts on beneficial insects and the environment. By carefully selecting and applying these pesticides, we aim to mitigate the negative effects on beneficial insects while effectively controlling diamondback moth infestations. This approach ensures a more sustainable and environmentally friendly pest management strategy. We aspire to make a meaningful contribution to the overall progress of effective and environmentally conscious integrated control strategies for managing diamondback moth infestations in cabbage production.

The rest of this paper is organized as follows: Section 2 elaborates the formulation and analysis of the model while Section 3 expounds upon the comprehensive global analysis of the model's parameters. In Section 4 we focused on discussions of the results. Finally, the concluding remarks is detailed in Section 5.

2. Model formulation

We proposed a system of non linear differential equations that capture the infestations dynamics of Diamondback moth on cabbage biomass by incorporating the inter coping and environmentally friendly pesticides. Our model system comprises of two populations: cabbage and *Plutella xylostella* The *Plutella xylostella* population is divided into four distinct developmental stages, namely Egg (E), Larvae (L), Pupae (P), and Adult moth (A). These stages represent different phases of the insect's life cycle, encompassing its growth and transformation from one form to another. The larva is considered to be an effective and destructive stage that affects the folliar tissues, leaves, and heads of the cabbages [36]. The cabbage is a seasonal vegetable harvested after 60–180 days of its growth [13]. We express the population density of cabbage biomass per plot as C, assuming that the cabbage planting takes place within a day at the start of the season.

Table 1
Descriptions of the model parameters.

Parameters	Descriptions of parameters
λ_L	Natural mortality rate of Larva
λ_P	Natural mortality rate of pupae
λ_E	Natural mortality rate of Eggs
α_E^{-1}	The average development time of the eggs
α_L^{-1}	Larval average growth time
α_P^{-1}	Pupal average growth time
λ_A^{-1}	An average life span of an adult female moth
K_C	The carrying capacity of the cabbage biomass
K_E	The carrying capacity of the Eggs
K_L	The carrying capacity of the Larva
η	Biomass conversion
ψ	Proportion of female adult moth
ω	Larvae cabbage attack rate
ϕ	Mortality rate of DBM at all stages due to intervention
q	The number of eggs laid per female DBM moth per day
r	Growth rate of cabbages

The developed model assumes the following: when food is scarce, larvae exhibit cannibalistic behavior on younger larvae [37]; there is no natural decay of cabbages since all advice from the farm experts and all the conditions for cabbage growth are considered; the Cabbage biomass grows at the rate r to its carrying capacity K_C. Let ω to be the DBM Caterpillar (larva) attack rate, η be the efficiency of biomass conversion and ϕ is used as a farmer's cropping methods (inter-cropping) and use of selected pesticides. The use appropriate farming methods are assumed to reduce DBM outbreak and impact in the cabbage field by a factor ϕ. This factor lies in the interval of $0 \leq \phi \leq 1$, and K_L is the environmental carrying capacity of DBM larvae, which is assumed to follow logistic growth.

The life cycle of *Plutella xylostella* begins with eggs, laid in clusters on the underside of the cabbage leaves. Following a duration of 2 to 9 days, the eggs of *Plutella xylostela* hatch into larvae. These larvae then proceed to burrow into the leaves, causing damage to the soft folliar tissues. After a period of 8 to 16 days, the larvae transition into the pupal stage. During this phase, the pupae no longer burrow into the leaves but instead feed on the external leaves. Basing on the environment conditions the pupae stage last for the duration of 5–15 days to turn to an adult moth [6].

We therefore propose the deterministic mathematical model to capture the dynamics of diamondback moth infestations in a cabbage biomass. Based on the flow diagram presented in Fig. 1, where the dotted red line represents interaction and biomass conversion. Also, ϕ represents the intervention strategy aiming at reducing the population of DBM in all stages in the cabbages biomass, resulting into establishment of non linear differential equations described in Eq. (1).

$$\begin{cases} \dfrac{dC}{dt} = r\left(1 - \dfrac{C}{K_C}\right)C - \omega LC, \\[2mm] \dfrac{dE}{dt} = \psi q\left(1 - \dfrac{E}{K_E}\right)A - (\lambda_E + \phi + \alpha_E)E, \\[2mm] \dfrac{dL}{dt} = \alpha_E\left(1 - \dfrac{L}{K_L}\right)E + \eta \omega LC - (\lambda_L + \phi + \alpha_L)L, \\[2mm] \dfrac{dP}{dt} = \alpha_L L - (\alpha_P + \phi + \lambda_P)P, \\[2mm] \dfrac{dA}{dt} = \alpha_P P - (\phi + \lambda_A)A, \end{cases} \quad (1)$$

with initial conditions $C(0) \geq 0, E(0) \geq 0, L(0) \geq 0, P(0) \geq 0, A(0) \geq 0$ (see Table 1).

2.1. Boundedness of the model solution

Theorem 1. *There is a unique solution to non-linear ordinary differential equation of model system Eq. (1) in $\mathbb{R}_+^5$ with the initial conditions of the model system Eq. (1). The solution remains within a region denoted as $\Omega = \{C, E, L, P, A\} \geq 0$ in the positive real numbers space $\mathbb{R}_+^5$.*
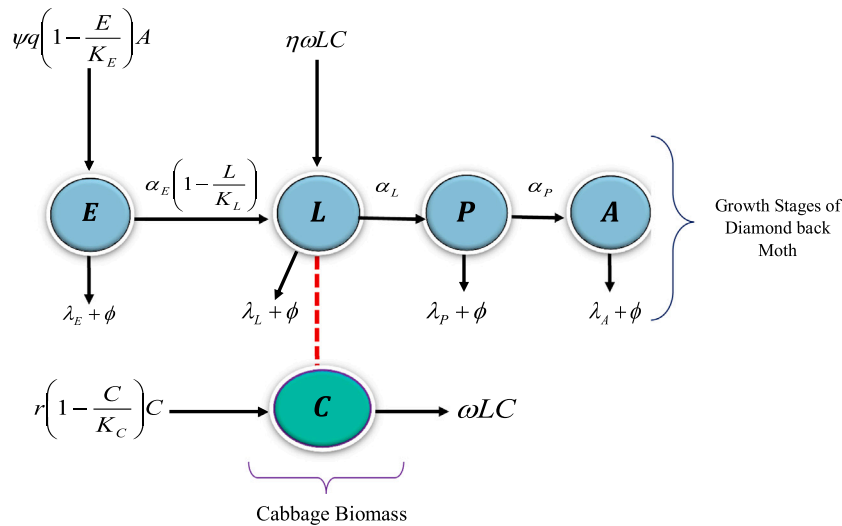
Fig. 1. The compartmental model diagram for the dynamics of DBM in a cabbage biomass.

The model consists of two populations: the cabbage biomass and the diamondback moth stage populations. To check the boundedness, we use the function Φ to represent the growth stage population of diamondback moth.

Proof. We let the function $\Phi = E + L + P + A$ then,

$$\frac{d\Phi}{dt} = \frac{dE}{dt} + \frac{dL}{dt} + \frac{dP}{dt} + \frac{dA}{dt}.$$
$$\leq A(\psi q + 1) - (\beta_1 L + \beta_2 P + \beta_3 A).$$
$$\leq m(\psi q + 1) - h\Phi, \tag{2}$$

for $m = \max\{E(0), M\}$ and $h = \min\{\beta_1, \beta_2, \beta_3\}$.

$$\frac{d\Phi}{dt} + h\Phi \leq m(\psi q + 1).$$

Solving Eq. (2) analytically we have the following:

$$\Phi(t) \leq \frac{m}{h}(\psi q + 1)\left(1 - e^{-ht}\right) + \Phi(0)e^{-ht}. \tag{3}$$

As $\lim_{t\to\infty} \Phi(t)$, then the solution of equation Eq. (2) becomes;

$$\Phi(t) \leq \frac{m}{h}(\psi q + 1). \tag{4}$$

The solution is bounded for $0 \leq \Phi(t) \leq \frac{m}{h}(\psi q + 1)$, and the solutions of the proposed system Eq. (1) are in the region $\Omega \in \mathbb{R}^5_+$ such that $\Phi(t) \leq \frac{m}{h}(\psi q + 1) + \epsilon, \forall \epsilon > 0, t \to 0$. Therefore, the model Eq. (1) is well posed in a region Ω. $\square$

2.2. Positivity of the model solution

By using positive initial data we check if the solutions of system Eq. (1) remains in $\mathbb{R}^5_+ \forall t > 0$.

Theorem 2. *Let the initial conditions of the model system Eq. (1) be satisfied, then the model solutions remain positive region $\Omega \in \mathbb{R}^5_+, \forall t \geq 0$.*

Proof. We consider first equation of the system Eq. (1), we establish that

$$\frac{dC}{dt} \leq rC\left(1 - \frac{C}{K_C}\right). \tag{5}$$

Upon solving Eq. (5) and algebraic simplifications we have Eq. (6)

$$C \leq \frac{K_C C(0)}{e^{-rt}\left(K_C - C(0)\right) + C(0)}. \tag{6}$$

As $\lim_{t\to\infty} C$, we get $0 \leq C \leq K_C$, Thus, all feasible solutions of the proposed model Eq. (1) remain in $\Omega \in \mathbb{R}^5_+$. Therefore, model is well-posed and it is now sufficient to check its dynamics. $\square$

2.3. Model equilibria

This subsection investigates the existence of equilibrium points in system Eq. (1). The system possesses four positive equilibrium points labeled as ξ_0, ξ_1, ξ_2, and ξ_3.

2.3.1. Equilibrium point ξ_0
The system Eq. (1) has a trivial equilibrium point $\xi_0 = (0, 0, 0, 0, 0)$. In the context of a farm, this equilibrium point suggests that in the absence of cabbages, pests are not drawn to the farm. This implies that the presence of cabbages acts as a primary attractant for the pests, and without this food source, the pests are not naturally inclined to infest the farm.

2.3.2. Pest-free equilibrium point ξ_1
The system Eq. (1) has a pest – free equilibrium point $\xi_1 = (0, 0, 0, 0, K_C)$. In the absence of the Diamondback moth population, the cabbage biomass can indeed grow to its carrying capacity.

2.3.3. Cabbage-free equilibrium point ξ_2
The system Eq. (1) has an equilibrium point $\xi_2 = (E_2, L_2, P_2, A_2, 0)$ such that,

$$E_2 = \frac{K_E K_L d_1 d_2 d_3 d_4 (\mathcal{R}_* - 1)}{K_E \alpha_E d_1 d_2 d_3 + \psi q \alpha_E \alpha_L \alpha_P K_L}, \quad L_2 = \frac{K_E K_L d_1 d_2 d_3 d_4 (\mathcal{R}_* - 1)}{\alpha_L \psi q \alpha_P (K_E \alpha_E + K_L d_2)},$$
$$P_2 = \frac{K_E K_L d_1 d_2 d_3 d_4 (\mathcal{R}_* - 1)}{\alpha_P \psi q (K_E \alpha_E + K_L d_2)}, \quad A_2 = \frac{K_E K_L d_1 d_2 d_3 d_4 (\mathcal{R}_* - 1)}{\psi q d_3 d_4 (K_E \alpha_E + K_L d_2)},$$

where: $d_1 = \alpha_E + \phi + \lambda_E$, $d_2 = \alpha_L + \phi + \lambda_L$, $d_3 = \alpha_P + \phi + \lambda_P$, $d_4 = \lambda_A + \phi$, and

$$\mathcal{R}_* = \frac{\psi q \alpha_E \alpha_L \alpha_P}{d_1 d_2 d_3 d_4}. \tag{7}$$

Ecologically, the growth of the DBM populations can have detrimental effects on cabbage biomass and potentially lead to the extinction of cabbages in the affected area. This is due to the destructive feeding habits of the DBM larvae, which consume cabbage heads and leaves. When faced with food scarcity, adult Diamondback Moth (DBM) have been observed to engage in cannibalistic behavior towards younger larvae as a survival strategy to acquire nourishment [37].

In the context of this paper, $\mathcal{R}_*$ described in (Eq. (7)), is defined as the average lifetime reproductive rate of a female diamondback moth (DBM). This reproductive rate represents the total number of offspring produced by a female DBM over the course of its entire lifespan.

If $\mathcal{R}_* > 1$, it signifies that the diamondback moth population will persist and thrive, leading to the destruction of cabbage leaves. This

$$J = \begin{pmatrix} r\left(1 - \dfrac{C}{K_C}\right) - \dfrac{rC}{K_C} - \omega L & 0 & -\omega C & 0 & 0 \\[2ex] 0 & -\dfrac{\psi q A}{K_C} - d_1 & 0 & 0 & \psi q\left(1 - \dfrac{C}{K_C}\right) \\[2ex] \eta\omega L & \left(1 - \dfrac{C}{K_C}\right)\alpha_L & \dfrac{\alpha_L E}{K_L} + \eta\omega C - d_2 & 0 & 0 \\[2ex] 0 & 0 & \alpha_L & -d_3 & 0 \\[2ex] 0 & 0 & 0 & \alpha_P & -d_4 \end{pmatrix} \tag{9}$$

Box I.

continuous infestation can ultimately result in the demise of cabbages, leading to their extinction.

2.3.4. Co-existence equilibrium point ξ_3

The system Eq. (1) has an equilibrium point $\xi_3 = \left(E_3, L_3, P_3, A_3, C_3\right)$ such that,

$$E_3 = \frac{q\psi\alpha_P K_E\left(\alpha_E g_3 + \sqrt{g_3^2 - 4g_1 g_2}\right)}{2K_E g_1 d_1 d_4 + \psi q\alpha_P\left(\alpha_L g_3 + \sqrt{g_3^2 - 4g_1 g_2}\right)},$$

$$L_3 = \frac{g_3 + \sqrt{g_3^2 - 4g_1 g_2}}{2g_1}, P_3 = \frac{\alpha_L\left(g_3 + \sqrt{g_3^2 - 4g_1 g_2}\right)}{2g_1 d_3},$$

$$A_3 = \frac{\alpha_P\left(\alpha_L g_3 + \alpha_E\sqrt{g_3^2 - 4g_1 g_2}\right)}{2g_1 d_1 d_4},$$

$$C_3 = \frac{K_C\left(2g_1 r + \eta\omega\left(g_3 + \sqrt{g_3^2 - 4g_1 g_2}\right)\right)}{2g_1};$$

where: $g_1 = \psi q\eta\omega K_L K_C\alpha_L\alpha_P$, $g_2 = (\eta\omega K_C + r)K_L K_E d_1 d_2 d_3$ and $g_3 = \psi q K_L K_C(\alpha_L\alpha_P(1 + r + d_2) + \eta\omega d_1 d_2 d_3)$.

Therefore, co-existence equilibrium point ξ_3 exists if condition Eq. (8) holds.

$$g_3^2 - 4g_1 g_2 \geq 0. \tag{8}$$

Theorem 3. *The Co-existence equilibrium point ξ_3 will exist if Eq. (8) holds.*

2.4. Local stability of the model equilibria

In this section, we examine the behaviors of the system Eq. (1) in the neighborhood of its equilibrium points. The Jacobian matrix (Eq. (9)) is computed for each equilibrium point. To assess the stability of each model equilibrium, we determined the eigenvalues of these matrices (see Box I).

2.4.1. Evaluating Jacobian matrix (9) at ξ_0

$$J(\xi_0) = \begin{pmatrix} r & 0 & 0 & 0 & 0 \\ 0 & \alpha_E - \lambda_E & 0 & 0 & 0 \\ 0 & \alpha_L & \alpha_L - \lambda_L & 0 & 0 \\ 0 & 0 & \alpha_L & -\lambda_P - \alpha_P & 0 \\ 0 & 0 & 0 & \alpha_P & -\lambda_A \end{pmatrix}. \tag{10}$$

From the matrix (10) we can clearly observe that one of the eigenvalues is positive, that is $\lambda_1 = r > 0$. Therefore, this confirms that ξ_0 is a saddle point.

2.4.2. Evaluating Jacobian matrix (9) at ξ_1

$$J(\xi_1) = \begin{pmatrix} -r & 0 & -\omega K_C & 0 & 0 \\ 0 & \alpha_E - \lambda_E & 0 & 0 & \psi q \\ 0 & \alpha_L & \eta\omega K_C - d_2 & 0 & 0 \\ 0 & 0 & \alpha_L & -d_4 & 0 \\ 0 & 0 & 0 & \alpha_P & -d_4 \end{pmatrix}. \tag{11}$$

Upon observing the matrix Eq. (11), we can identify that one of the eigenvalues is negative, specifically $\lambda_1 = -r < 0$. The remaining eigenvalues are obtained from the characteristic polynomial presented in Eq. (12):

$$\lambda^4 + k_1\lambda^3 + k_2\lambda^2 + k_3\lambda + k_4 = 0 \tag{12}$$

where,
$k_1 = d_1 + d_2 + d_4 - \eta\omega K_C.$
$k_2 = (d_1 + d_2)(d_3 + d_4) + d_1 d_2 + d_3 d_4 - \eta\omega(d_1 + d_3 + d_4).$
$k_3 = d_1(d_2 d_4 + d_2(d_3 + d_4)) + d_2 d_3 d_4 - \eta\omega K_C(d_1(d_3 + d_4) + d_3 d_4).$
$k_4 = d_1 d_2 d_3 d_4 - \eta\omega K_C - \psi q\alpha_E\alpha_L\alpha_P.$
The Routh Hurwitz Criteria for stability of ξ_1 are;

$$\begin{cases} M_1 = k_1 > 0; k_3 > 0; k_4 > 0 \\ M_2 = k_1 k_2 k_3 - k_3^2 - k_1^2 k_4 > 0 \end{cases} \tag{13}$$

Theorem 4. *A point ξ_1 will be classified as locally asymptotically stable if all the conditions described in Eq. (13) are fulfilled. Conversely, if any of these conditions are not met, the point ξ_1 will be deemed as unstable node.*

2.4.3. Local stability for ξ_2

We have to evaluated the Jacobian matrix (9) at ξ_2 which gives

$$J(\xi_2) = \begin{pmatrix} r - w & 0 & 0 & 0 & 0 \\ 0 & -y & 0 & 0 & f \\ 0 & \hat{k} & \hat{m} & 0 & 0 \\ 0 & 0 & \alpha_L & -d_3 & 0 \\ 0 & 0 & 0 & \alpha_P & -d_4 \end{pmatrix} \tag{14}$$

where,

$$w = \frac{K_E K_L\omega d_1 d_2 d_3 d_4\left(\mathscr{R}_* - 1\right)}{(K_E\alpha_E + K_L d_1 d_2)\psi q\alpha_L\alpha_P}, \hat{m} = \frac{\psi q\alpha_L\alpha_P(K_E\alpha_E + K_L d_1 d_2)}{K_E d_1 d_3 d_4 + \psi q K_L\alpha_L\alpha_P},$$

$$f = \frac{\psi d_1 d_4(K_E\alpha_E + K_L d_2)}{K_E\alpha_E d_1 d_3 d_4 + \psi q\alpha_E\alpha_L\alpha_P},$$

$$\hat{k} = \frac{K_E d_1 d_3 d_4}{\psi q\alpha_L\alpha_P(K_E\alpha_E + K_L d_1 d_2)},$$

$$y = \frac{-d_1 d_3 d_4\left(2K_L d_2 + K_E\alpha_E\right) + \psi q\alpha_E\alpha_L\alpha_P}{d_3 d_4(K_E\alpha_E + K_L d_3)}.$$

Observing the matrix (14) we see that one of the eigenvalues is negative, specifically $\lambda_1 = r - w < 0$ iff $w > r$ and $\mathscr{R}_* > 1$ the remaining eigenvalues are obtained from the characteristic polynomial Eq. (15) given by

$$\lambda^4 + c_1\lambda^3 + c_2\lambda^2 + c_3\lambda + c_4 = 0 \tag{15}$$

where,

$c_1 = d_1 + d_2 + d_4 - \eta \omega K_C,$

$c_2 = (d_1 + d_2)(d_3 + d_4) + d_1 d_2 + d_3 d_4 - \eta \omega (d_1 + d_3 + d_4),$

$c_3 = d_1(d_2 d_4 + d_2(d_3 + d_4)) + d_2 d_3 d_4 - \eta \omega K_C(d_1(d_3 + d_4) + d_3 d_4),$

$c_4 = d_1 d_2 d_3 d_4 - \eta \omega K_C - \psi q \alpha_E \alpha_L \alpha_P.$

The Routh Hurwitz Criteria for stability of ξ_2 are;

$$\begin{cases} N_1 = c_1 > 0; c_3 > 0; c_4 > 0 \\ N_2 = c_1 c_2 k_3 - c_3^2 - c_1^2 c_4 > 0 \end{cases} \quad (16)$$

If $\lambda_1 = r - w < 0$ iff $w > r$, $\mathscr{R}_* > 1$ and the conditions stated in Eq. (16) are satisfied and $\mathscr{R}_* > 1$ then ξ_2 is locally asymptotically stable point.

2.4.4. Local stability for co-existence equilibrium point ξ_3

Evaluating Jacobian matrix (9) at ξ_3 gives,

$$J(\xi_3) = \begin{pmatrix} h_1 & 0 & h_2 & 0 & 0 \\ 0 & h_3 & 0 & 0 & h_4 \\ h_5 & h_6 & h_7 & 0 & 0 \\ 0 & 0 & \alpha_L & -d_3 & 0 \\ 0 & 0 & 0 & \alpha_P & -d_4 \end{pmatrix}, \quad (17)$$

where;

$h_1 = r - \frac{2rC}{K_C} - \omega, \quad h_2 = -\omega C, h_2 = -\omega C, h_3 = d_1 - \frac{\psi q A}{K_E}, \quad h_4 = \psi q\left(1 - \frac{E}{K_E}\right), \quad h_5 = \eta \omega L, \quad h_6 = \alpha_E\left(1 - \frac{L}{K_L}\right), \quad h_7 = \eta \omega C - d_2 - r - \frac{\alpha_L E}{K_L}.$

We find the coefficients of the characteristic polynomial Eq. (18) of the matrix (17). Basing on the Routh–Hurwitz criteria these coefficients will be used to conclude on the stability of ξ_3.

$$\lambda^4 + b_1 \lambda^3 + b_2 \lambda^2 + b_3 \lambda + b_4 = 0. \quad (18)$$

The coefficients are; $b_1 = d_3 + d_4 - (h_1 + h_7),$

$b_2 = h_1(h_3 + h_7) + h_2 h_5 - h_3 h_7 + (d_4 - 2h_1 + 2h_3 - 2h_7)d_3,$

$b_3 = d_3(h_5 h_2 - h_3 h_7 + h_1 h_7 - h_1 h_3) + h_3(h_1 h_7 + h_2 h_5) - d_3(h_1 h_3 - h_1 h_7 + d_4(h_1 h_3 + h_7) - h_2 h_5),$

$b_4 = d_3 h_3(h_2 h_5 + h_1 h_7) + d_3(h_3(h_2 h_5 - h_7 h_1) - d_4(h_2 h_5 - h_3 h_7 + h_1(h_7 - h_3))) - h_4 h_6 \alpha_L \alpha_P,$

$b_5 = \alpha_L \alpha_P h_6 h_4 h_1 + h_3 d_3 d_4(h_2 h_5 + h_1 h_7).$

The Routh–Hurwitz criteria for fifth-degree polynomials determines the local stability of equilibrium points [38,39]. Therefore, if the conditions stated in (19) are satisfied then equilibrium point ξ_3 gains stability.

$$\begin{cases} S_1 = b_1 > 0; b_3 > 0; S_2 = b_1 b_2 - b_3 > 0; \\ S_3 = b_1 b_2 b_3 - b_4 - b_1^2 - b_3^2 > 0; \\ S_4 = (b_3 b_4 - b_2 b_5)(b_1 b_2 - b_3) - (b_1 b_4 - b_5)^2 > 0; \\ S_5 = b_5 S_4 > 0 \end{cases} \quad (19)$$

The coexistence equilibrium point ξ_3 will exhibit local asymptotic stability as a stable node if the conditions stated in (19) holds.

2.5. The global analysis of model equilibria

In this section we analyze the global behaviors of the model equilibria by using Lyapunov function.

2.5.1. Trivial equilibrium point ξ_0

Theorem 5. *To ensure global asymptotic stability of the equilibrium point ξ_0, it is necessary and sufficient that $C = K_C$, $\mathscr{R}_* \leq 1$, and $d_1 d_4 \eta \omega \leq \psi q \omega \alpha_E$.*

Proof.

$$\mathcal{V}_0(t) = C + \left(\frac{d_4}{q\psi}\right)E + \left(\frac{d_1 d_4}{q\psi \alpha_E}\right)L + \left(\frac{d_1 d_2 d_4}{q\psi \alpha_E \alpha_L}\right)P + \left(\frac{d_1 d_2 d_3 d_4}{q\psi \alpha_E \alpha_L \alpha_P}\right)A. \quad (20)$$

The Lyapunov candidate function Eq. (20) is properly structured, continuous, and positive definite $\forall$ C, E, L, P, and A. It is evident that, $\mathcal{V}_0$ perish at ξ_0. Taking the derivatives of the Eq. (20) and plugging the derivatives from the model system Eq. (1). we have,

$$\mathcal{V'}_0(t) = C' + \left(\frac{d_4}{q\psi}\right)E' + \left(\frac{d_1 d_4}{q\psi \alpha_E}\right)L' + \left(\frac{d_1 d_2 d_4}{q\psi \alpha_E \alpha_L}\right)P'$$

$$+ \left(\frac{d_1 d_2 d_3 d_4}{q\psi \alpha_E \alpha_L \alpha_P}\right)A',$$

$$= rC\left(1 - \frac{C}{K_C}\right) - \omega LC + \left(\frac{d_4}{q\psi}\right)\left(q\left(1 - \frac{E}{K_E}\right)\psi A - d_1 E\right)$$

$$+ \left(\frac{d_1 d_4}{q\psi \alpha_E}\right)\left(\alpha_E\left(1 - \frac{L}{K_L}\right)E + \eta \omega LC - d_2 L\right)$$

$$+ \left(\frac{d_1 d_2 d_4}{q\psi \alpha_E \alpha_L}\right)(\alpha_L L - d_3 P) + \frac{d_1 d_2 d_3 d_4 P}{q\psi \alpha_E \alpha_L \alpha_P} + \frac{d_1 d_2 d_3 d_4^2 A}{q\psi \alpha_E \alpha_L \alpha_P}.$$

$$= rC\left(1 - \frac{C}{K_C}\right) - \omega LC + \left(\frac{d_4}{q\psi}\right)\left(q\left(1 - \frac{E}{K_E}\right)\psi A - d_1 E\right)$$

$$+ \left(\frac{d_1 d_4}{q\psi \alpha_E}\right)\left(\alpha_E\left(1 - \frac{L}{K_L}\right)E + \eta \omega LC - d_2 L\right)$$

$$+ \left(\frac{d_1 d_2 d_4}{q\psi \alpha_E \alpha_L}\right)(\alpha_L L - d_3 P) + \frac{d_1 d_2 d_3 d_4 P}{q\psi \alpha_E \alpha_L \alpha_P} + \frac{d_1 d_2 d_3 d_4^2 A}{q\psi \alpha_E \alpha_L \alpha_P}$$

$$= -d_4 \frac{EA}{K_E} - d_1 d_2 \frac{EL}{K_L} - \frac{d_1 d_2 d_3 d_4^2}{q\psi \alpha_E \alpha_L \alpha_P}\left(1 - \frac{q\psi \alpha_E \alpha_L \alpha_P}{d_1 d_2 d_3 d_4}\right)A$$

$$- \eta \omega\left(\frac{q\psi \alpha_E \omega}{d_1 d_4 \eta \omega} - 1\right)LC + rC\left(1 - \frac{C}{K_C}\right)$$

$$\mathcal{V}_0(t) = -d_4 \frac{EA}{K_E} - d_1 d_2 \frac{EL}{K_L} - \frac{d_4}{\mathscr{R}_*}(1 - \mathscr{R}_*)A - \eta \omega\left(\frac{q\psi \alpha_E \omega}{d_1 d_4 \eta \omega} - 1\right)LC$$

$$+ rC\left(1 - \frac{C}{K_C}\right). \quad (21)$$

Therefore, from Eq. (21), $\mathcal{V}_0(t) = 0$ if and only if $C = K_C$, $\mathscr{R}_* = 1$, and $d_1 d_4 \eta \omega \leq q\psi \alpha_E \omega$. Then $\mathcal{V}_0(t)$ is negative definite if $C = K_C$, $\mathscr{R}_* \leq 1$, and $d_1 d_4 \eta \omega \leq q\psi \alpha_E \omega$. $\square$

2.5.2. Equilibrium point ξ_1

To check the global stability of this point we formulate good lyapunov candidate that will be used to perform the analysis.

Theorem 6. *Pest free-equilibrium point ξ_2 ensures global asymptotic stability if (i) $\mathscr{R}_* \leq 1$, (ii) $C < C_*$, and (iii) $\omega q\psi \alpha_E C_* \leq \left(d_1 d_4 \eta \omega + \omega q\psi \alpha_E C\right)$ are true.*

Proof. We let the Lyapunov function described in Eq. (22)

$$\mathcal{V}_1(t) = C - C_* - C_* \ln\left(\frac{C}{C_*}\right) + \left(\frac{d_4}{q\psi}\right)E + \left(\frac{d_1 d_4}{q\psi \alpha_E}\right)L + \left(\frac{d_1 d_2 d_4}{q\psi \alpha_E \alpha_L}\right)P$$

$$+ \left(\frac{d_1 d_2 d_3 d_4}{q\psi \alpha_E \alpha_L \alpha_P}\right)A. \quad (22)$$

This is evident that lyapunov candidate function (22) is well defined, continuous, and positive definite $\forall$ C, E, L, P, and A. $\mathcal{V}_1$ perish at ξ_1,
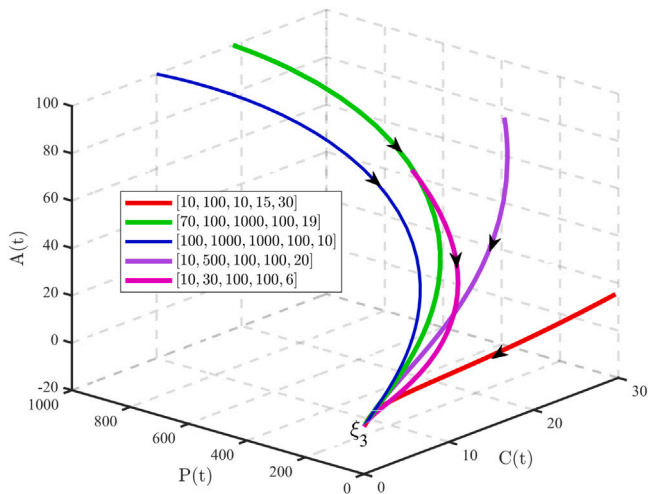
Fig. 2. Illustrates non linear stability analysis of point ξ_3 on P–C–A plane.

Table 2
Numerical values of the parameters.

Parameters	Value	Source
λ_L	0.1500	Faithpraise et al. [40].
λ_P	0.3000	Faithpraise et al. [40].
λ_E	0.3700	Faithpraise et al. [40]
α_E^{-1}	7 (2–9) days	[37].
α_L^{-1}	14 (8–16) days	[6,37].
α_P^{-1}	10 (5–15) days	[6,37].
λ_A^{-1}	18 (15–18) days	[6,37].
K_C	30 Leaves Plant^{-1}	Tonnang et al. [9],Faithpraise et al. [40].
K_E	10^6	Estimate.
K_L	10^6	Estimate.
η	0.02 days^{-1}	Estimate.
ψ	0.07 days^{-1}	Estimate.
ω	6×10^{-6} days^{-1}	Estimate.
ϕ	Varied	
q	160	[37].
r	0.05	Faithpraise et al. [40].

Consequently, $\mathcal{V}'_1(t)$ along the solutions of system (1) holds.

$$\mathcal{V}'_1(t) \leq \left(1 - \frac{C_*}{C}\right)C' + \left(\frac{d_4}{q\psi}\right)E' + \left(\frac{d_1 d_4}{q\psi \alpha_E}\right)L' + \left(\frac{d_1 d_2 d_4}{q\psi \alpha_E \alpha_L}\right)P'$$
$$+ \left(\frac{d_1 d_2 d_3 d_4}{q\psi \alpha_E \alpha_L \alpha_P}\right)A'$$
$$= -rC_*\left(1 - \frac{C}{K_C}\right)\left(1 - \frac{C}{C_*}\right) - d_4 \frac{EA}{K_E} - d_1 d_2 \frac{EL}{K_L}$$
$$- \frac{d_1 d_2 d_3 d_4^2}{q\psi \alpha_E \alpha_L \alpha_P}(1 - \mathcal{R}_*)A$$
$$- \frac{\omega q\psi \alpha_E + d_1 d_4 \eta \omega}{q\psi \alpha_E}\left(1 - \frac{\omega q\psi \alpha_E C_*}{(d_1 d_4 \eta \omega + \omega q\psi \alpha_E)C}\right).$$

This conclude that, $\mathcal{V}_1$ is negative definite if the conditions in Eq. (23) are true.

$$\mathcal{R}_* \leq 1, \ C < C_* \text{ and } \omega q\psi \alpha_E C_* \leq (d_1 d_4 \eta \omega + \omega q\psi \alpha_E). \quad \square \quad (23)$$

2.5.3. Co-existence equilibrium point ξ_3

The stability of ξ_3 was assessed by plotting three-dimensional graphs. The figures presented in Figs. 2 and 3 demonstrate that the trajectories of the system converge towards the equilibrium point ξ_3 in the invariant region Ω. Therefore, it can be concluded that ξ_3 is an asymptotically stable point.

3. Global sensitivity analysis

In this section, we determine the model's response to variations in each parameter within an uncertainty range using the parameter baseline values presented in Table 2. Utilizing Latin hypercube sampling approach, we obtain the partial rank correlation coefficients (PRCC). Based on the PRCC indices, we observe that the parameters α_E, α_L, ψ, and α_P have strong negative correlations with cabbage biomass, as indicated in Fig. 6. Conversely, parameter ϕ has a strong positive correlation with cabbage biomass. Furthermore, the parameters α_E, α_L, and α_P exhibit a strong positive correlation with the stages of the diamondback moth (DBM), as shown in Figs. 4 and 5. Hence, to enhance cabbage production, it is advisable to implement a control strategy that primarily targets the management of the sensitive growth parameters of DBM, namely α_E, α_L, and α_P. By decreasing the values of these parameters, it becomes possible to suppress the population of DBM within the cabbage biomass.

4. Results and discussions

In this section, we perform simulations to study pest infestations as described in the model system Eq. (1). The results depicted in Fig. 7 demonstrate the outbreak and the impact of the DBM on cabbage biomass with initial values of $E(0) = 1000$, $L(0) = 100$, $P(0) = 100$, $A(0) = 1000$, and parameter values in Table 2. We observed that the cabbage biomass is declining due to the persistence of DBM in the cabbage farm. The analytical results depicted in Fig. 8 demonstrate the infestation of DBM in the cabbage biomass, with a threshold value of $\mathcal{R}_* < 1$. This indicates that after a period of 200 days, if a female DBM does not produce additional offspring, the population of DBM will eventually go extinct. Meanwhile, the cabbage population continues to grow over time, reaching its expected carrying capacity of 30 leaves per plant.

By setting the control parameter $\phi > 0.5$, it ensures that $\mathcal{R}_* < 1$. For instance, in Fig. 8, when $\phi = 0.9$, we obtain $\mathcal{R}_* = 0.3070$, which implies that an adult female moth is unable to produce more than one offspring. Similarly, by setting the control parameters $\phi < 0.5$, it guarantees that $\mathcal{R}_* > 1$. This indicates that the pest population persists by producing more offspring within the cabbage biomass, and once they reach the larval stage, they begin to attack and destroy the foliar tissues of the cabbage plants. For instance, when we set $\phi = 0.45$ and $\phi = 0.4$, the corresponding threshold numbers become $\mathcal{R}_* = 1.4630$ and $\mathcal{R}_* = 6.7722$, respectively.

5. Conclusion

In this paper, we have developed a deterministic mathematical model to capture the infestation dynamics of *Plutella xylostella* (Diamondback Moth, DBM) in a cabbage farm. The main objective of this study was to propose an intervention strategy that minimizes the threshold number and controls the moth populations. Numerical simulations were conducted to support the analytical results and address the persistence of pests within the cabbage biomass. The numerical findings, based on the system Eq. (1), indicate that when the threshold number $\mathcal{R}_* < 1$, the pest population in the cabbage field decreases significantly, leading to an increase in cabbage production up to its carrying capacity of 30 leaves per plant. The numerical simulations depicted in Fig. 7 demonstrate the detrimental impact of DBM outbreaks on the cabbage biomass, illustrating how the moth persists and destroys the cabbage plants.

To address and mitigate DBM outbreaks, we propose implementing intervention strategies as outlined in Fig. 8. Specifically, in Fig. 8(a), it is evident that increasing the value of ϕ results in an increase in cabbage biomass up to its carrying capacity, which is $K_C = 30$ leaves per plant. This suggests that by adjusting the control parameter ϕ, we
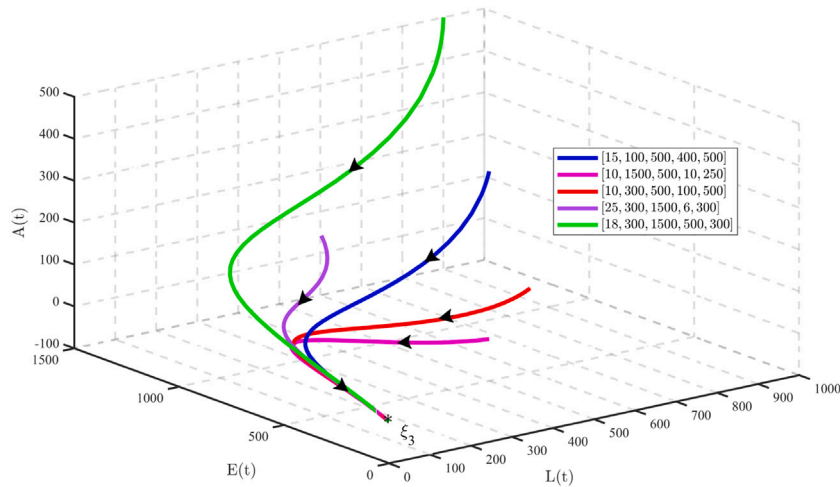
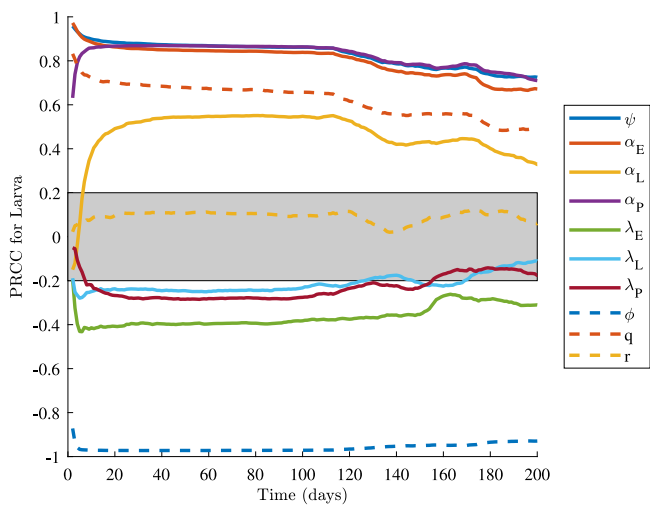**Fig. 3.** Illustrates non linear stability analysis of point ξ_3 on *E–L–A* plane.

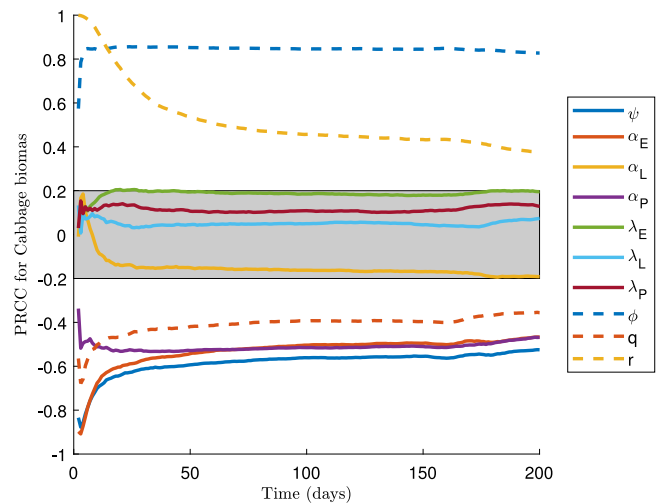**Fig. 4.** Global sensitivity analysis of parameters with respect to larvae stage.

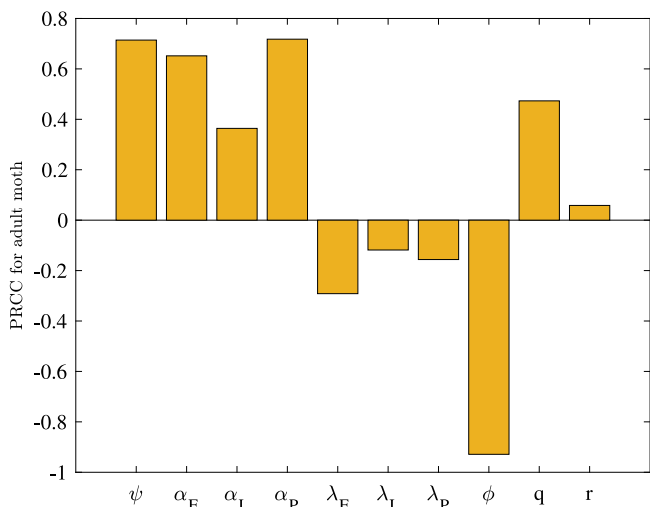**Fig. 6.** Demonstrates the variation of PRCC indices of parameters with respect to Cabbage biomass.

Fig. 5. Global sensitivity analysis of parameters with respect to adult moth stage.

can effectively manage and enhance cabbage production while minimizing the impact of DBM infestations. Additionally, we suggest the application of selected insecticides that have minimal negative impact on ecosystems and beneficial insects, as this can effectively reduce the pest population in the cabbage farm. Overall, this study contributes to our understanding of the infestation dynamics of DBM in cabbage farms and provides insights into potential intervention strategies for managing and minimizing pest populations.

Future research directions: We plan to investigate the effects of seasonality on the dynamics of *Plutella xylostella* on cabbages. The data obtained from the literature was not sufficient for calibrating and validating the model. We will perform these tasks once we have access to real data in the future. Furthermore, the developed deterministic differential model will be used to solve the fractional order delayed differential models [41,42], delayed dynamical models [43,44], and nonlinear systems of differential equations [45,46].
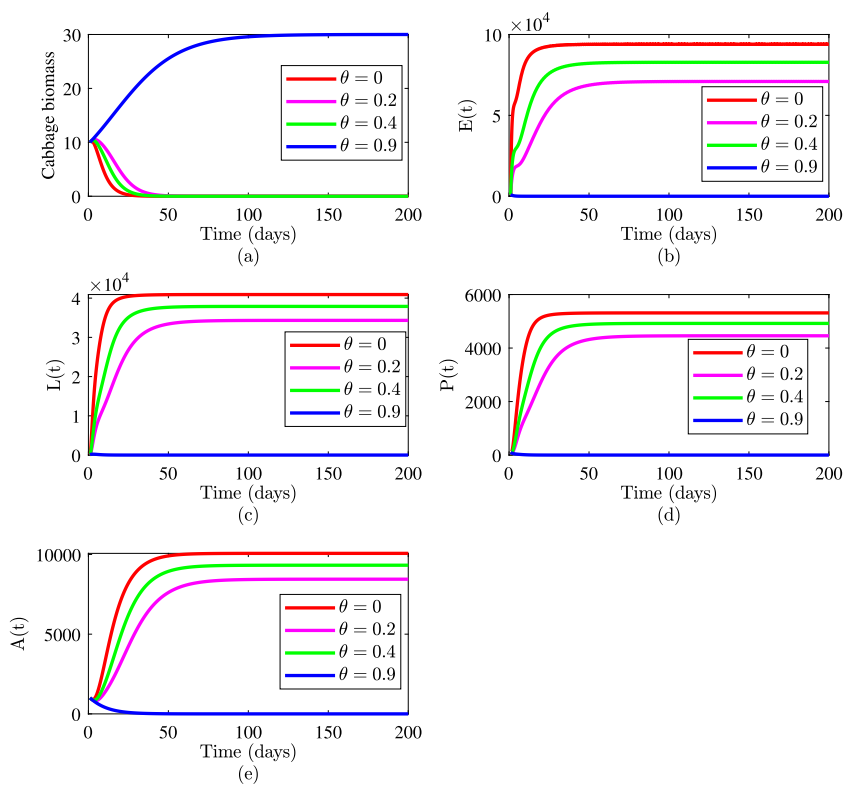
Fig. 7. Illustrates the impact of larger invasion of DBM populations in a cabbage biomass with varying values of ϕ.
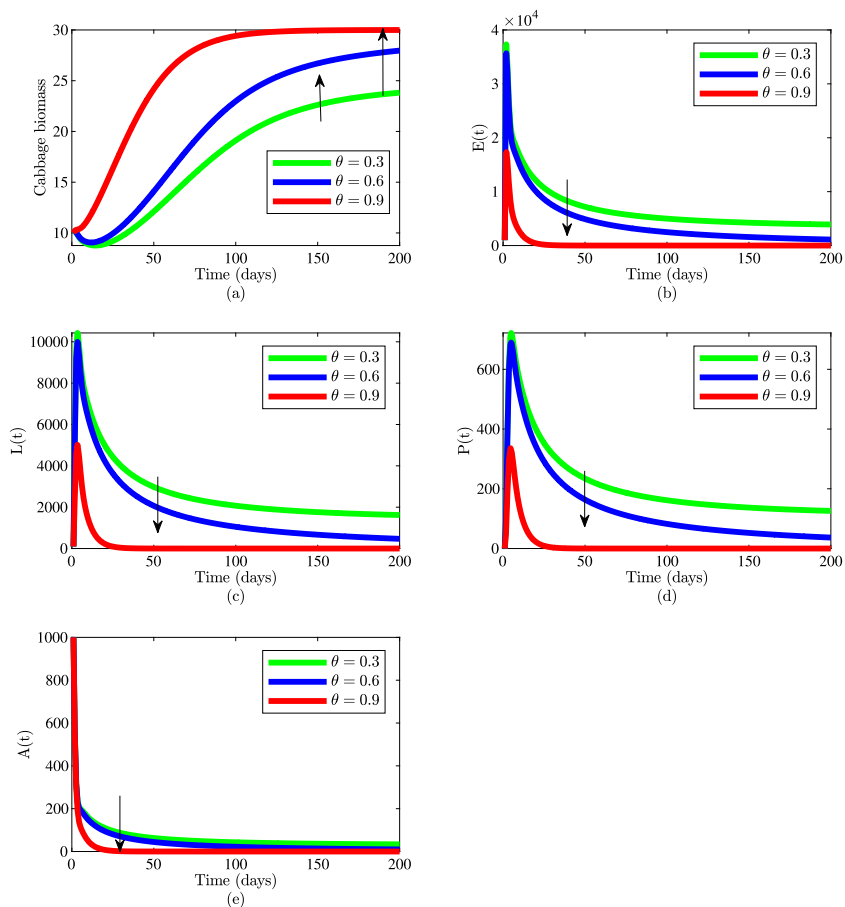

Fig. 8. Demonstrates the impact of a DBM outbreak in a cabbage biomass with a small initial pest population, considering varying values of the intervention strategy.

Declaration of competing interest

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